Page: Lec «2» Date: 19/3/2014

Inverse L.T

Ex: $L^{-1}\left(\frac{6}{s}\right) = 6L^{-1}\left(\frac{1}{s}\right) = 6$

Ex: $L^{-1}\left(\frac{3}{s-6}\right) = 3L^{-1}\left(\frac{1}{s-6}\right) = 3e^{6t}$

Ex: $L^{-1}\left(\frac{s+10}{s^2-16}\right) = L^{-1}\left[\frac{s}{s^2-16} + \frac{10}{s^2-16}\right]$

$= \cosh 4t + \frac{10}{4}\sinh 4t$

Ex: $L^{-1}\left[\frac{1}{s^2+6s+25}\right] = \frac{1}{4}L^{-1}\left[\frac{4}{(s+3)^2+16}\right]$

$= \frac{1}{4}e^{-3t}\sin 4t$

Ex: $L^{-1}\left[\frac{1}{(s-1)(s+2)(s-4)}\right] = L^{-1}\left[\frac{A}{s-1} + \frac{B}{s+2} + \frac{C}{s-4}\right]$

$A|_{s=1} = -1/9$, $B|_{s=-2} = 1/18$, $C|_{s=4} = 1/18$

$Ae^{t} + Be^{-2t} + Ce^{4t}$ ← لو الوقت ملخاش أحلها

Ex: $L^{-1}\left[\frac{s+2}{s^2+2s+3}\right] = L^{-1}\left[\frac{s+\textcircled{2}}{(s+1)^2+2}\right]$ → 1+1

$= L^{-1}\left[\frac{(s+1)+1}{(s+1)^2+2}\right] = L^{-1}\left[\frac{s+1}{(s+1)^2+2} + \frac{1}{(s+1)^2+2}\right]$

$= e^{-t}\left[\cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t\right]$

Ex: $L^{-1}\left[\underbrace{\ln\frac{s+2}{s+1}}_{F(s)}\right] =$ ⇝ (هامة جداً)

$= L^{-1}[\ln(s+2) - \ln(s+1)$

$F'(s) = \frac{1}{s+2} - \frac{1}{s+1}$

$L^{-1}[F's] = e^{-2t} - e^{-t} = -tP(t)$

$\therefore P(t) = \frac{e^{-t} - e^{-2t}}{t}$

قانون: $L^{-1}[F'(s)] = -tP(t)$

Ex: $L^{-1}\left[\frac{2s-1}{s^3-5}\right] = L^{-1}\left[\frac{2s-1}{5(s^2-1)}\right] - L^{-1}\left[\frac{2s-1}{5(s-1)(s+1)}\right]$

قاعدة $\Rightarrow L(y'(t)) = \int_0^\infty e^{-st}\, y'(t)\, dt$

$\int u\,dv = uv - \int v\,du$ $\quad u \downarrow \quad \downarrow dv$

$du = -s e^{-st}\, dt$, $v = y(t)$

$= e^{-st}\cdot y(t)\Big|_0^\infty + s\int_0^\infty e^{-st}\, y(t)\, dt$

$\therefore L(y'(t)) = -y(0) + s\,y(s)$

$$L(y'(t)) = s\,y(s) - y(0)$$

قاعدة هامة $\Rightarrow L^{-1}\left(\frac{F(s)}{s}\right) = \int_0^t f(t)\, dt$

$= L\left[\int_0^t f(t)\, dt\right] = \frac{F(s)}{s}$

← القانون ده لأى دالة مقسومة ÷ s

Ex: $L\int_0^t \sin 2u\, du = \frac{1}{s}\left(\frac{2}{s^2+4}\right)$ #

Ex: $L^{-1}\left[\frac{1}{s(s^2+1)}\right] = L^{-1}\left[\frac{A}{s} + \frac{Bs+C}{s^2+1}\right]$

حل آخر $\Rightarrow L^{-1}\left[\frac{\frac{1}{s^2+1}}{s}\right] = L^{-1}\left[\frac{F(s)}{s}\right]$

$= \int_0^t \sin t\, dt = -\cos t\Big|_0^t = 1 - \cos t$

Page: Date: / /

Ex:- $L^{-1}\left[\frac{1}{s^3(s+1)}\right] = L^{-1}\left[\frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{D}{s+1}\right]$

سهل ولكن لكله طويل ، في طريقة تانية اسهل

$= L^{-1}\left[\frac{1}{s}\cdot\frac{1}{s}\cdot\frac{1}{s}\cdot\underbrace{\frac{1}{s+1}}_{F(s)}\right]$

$= L^{-1}\left[\frac{1}{s}\cdot\frac{1}{s}\int_0^t e^{-t}\,dt - e^{-t}\Big|_0^t\right]$

$t + e^{-t}\Big|_0^t = t + e^{-t} - 1$

في المسألة دي لو حطيها بالتوالت وجبتش فيها التوالت هتبقى بقى الرجيم عشان هي اصلا ليها قاعدة تانية

Convolution therom

$$f(t) \overset{\text{Convolution}}{*} g(t) = \int_0^t f(x)\cdot g(t-x)\,dx$$

Ex:- $f(t) = e^{2t}$, $g(t) = e^{3t}$

$$\Rightarrow f*g = \int_0^t e^{2x}\cdot e^{3(t-x)}\,dx$$

$$= \int_0^t e^{3t}\,e^{-x}\,dx = e^{3t}\cdot e^{-x}\Big|_0^t = e^{3t}(1-e^{-t})$$

$$\therefore f*g = e^{3t}(1-e^{-t})$$

$-e^{-t}\Big|_0^t$

$-(e^{-x}+1)$

- $f*g = g*f$
- $L[f(t)*g(t)] = F(s)\cdot G(s)$
- $L^{-1}[F(s)\cdot G(s)] = f(t)*g(t)$

Page: Date: / /

Ex. $L^{-1}\left[\frac{1}{(s-1)(s+3)}\right] = L^{-1}\left[\underbrace{\frac{1}{s-1}}_{F(s)} \cdot \underbrace{\frac{1}{s+3}}_{G(s)}\right]$

$f(t) = e^t$, $g(t) = e^{-3t}$

$[\quad] = e^t * e^{-3t} = \int_0^t e^x \cdot e^{-3(t-x)}\, dx$

Ex:- $L^{-1}\left[\frac{s^2}{(s^2+4)^2}\right] = L^{-1}\left[\underbrace{\frac{s}{s^2+4}}_{F(s)\ (\cos 2t)} \cdot \underbrace{\frac{s}{s^2+4}}_{G(s)\ (\cos 2t)}\right]$

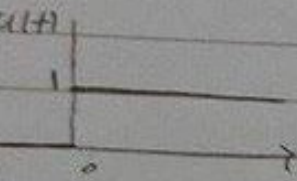
$= \cos 2t * \cos 2t$

$= \int_0^t \cos 2x\, (\cos 2(t-x))\, dx$

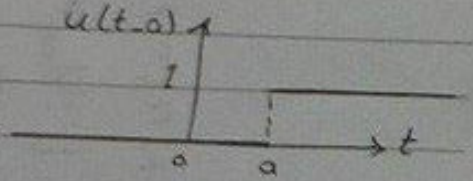
ملحوظة ⇐ أى مسألة تحل بال Convolution هى تحل بالكسور الجزئية

Unit Step Function

* $u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$



* $u(t-a) = \begin{cases} 0 & t < a \\ 1 & t \geq a \end{cases}$



* $L(u(t-a)) = \int_0^\infty e^{-st} \cdot u(t-a)\, dt$

$$\boxed{L(u(t-a)) = \frac{e^{-as}}{s}} \Rightarrow L^{-1}\left[\frac{e^{-as}}{s}\right] = u(t-a)$$

Page: Date: / /

"Summary" ملخص المحاضرة

$\mathcal{L}^{-1}$. T

- الجدول
- كسور جزئية
- إكمال مربع ⇓ "Shift therom" $\mathcal{L}^{-1}[F(s-a)] = e^{at} f(t)$
- $\mathcal{L}^{-1}\left[\frac{F(s)}{s}\right]$ → كسور جزئية / $\int_0^t f(t)\,dt$
- ln (tan⁻¹, cot⁻¹, ...) / $\mathcal{L}^{-1}[F'(s)] = -t f(t)$
- $\mathcal{L}^{-1}\left[\frac{F(s)}{(\,)(\,)}\right]$ → كسور جزئية / Convolution $f * g$
- $\mathcal{L}^{-1}[e^{-as} F(s)] = f(t-a)\,u(t-a)$

Report (1)

[1] Solve Applicat — L.T, Inverse L.T

(2) L.T Periodic f^{n}

$f(x+T) = f(x)$, periodic with Period $= T$

$L|\sin t|$